We consider a dynamic market model where buyers and sellers submit limit orders. If at a given moment in time, the buyer is unable to complete his entire order, the unmatched part of the order is recorded in the order book. Subsequently these buy unmatched orders may be matched with new incoming sell orders. The resulting demand curve constitutes the sole input to our model. The clearing price is then calculated using the market clearing condition. We use a Brownian sheet to model the demand curve, and provide some theoretical assumptions under which such a model is justified.

We prove that if there exists a unique equivalent martingale measure for the clearing price, then under some mild assumptions there is no arbitrage. We use the Ito-Wentzell lemma to obtain that result, and to characterize the dynamics of the demand curve and of the clearing price in the equivalent measure. We find that the volatility of the clearing price is (up to a stochastic factor) inversely proportional to the sum of buy and sell order flow density (evaluated at the clearing price). We also demonstrate that our approach is implementable. We use real order book data and simulate option prices under a particularly simple parameterization of our model. (Received September 21, 2012)