Can one count the number of critical points for random smooth functions of many variables? How complex is a typical random smooth function? How complex is the topology of its level sets? We study here the simplest case of smooth Gaussian random functions defined on the sphere in high dimensions. We show that such a randomly chosen smooth function is very complex, i.e. that its number of critical points of given index is exponentially large. We also study the topology of the level sets of these functions, and give sharp estimates of their Euler characteristic. This study, which is a joint work with Tuca Auffinger (Chicago) and partly with Jiri Cerny (Vienna), relies rather surprisingly on Random Matrix Theory. The main motivation comes from the study of energy landscapes for notoriously hard problems of statistical mechanics of disordered media, i.e. general spherical spin-glasses. I will detail the interesting picture we get for the complexity of these random Hamiltonians, and for the bottom of the energy landscape. We also propose a new invariant for the possible transition between two very different classes of complexity. (Received September 20, 2012)