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*Solving an Ill Conditioned Linear System using the Extended Iterative Refinement Algorithm: The Convergence Theorem.*

Many numerical analysts assume the convergence of the (extended) iterative refinement algorithm. We consider the linear system  $CW = U$  where  $W$  is the unknown matrix. We use the following extended iterative refinement algorithm to solve for  $W$ :

$$\begin{aligned}W_0 &= C_0^{-1}U_0 \quad (U_0 = U \text{ and } C_0 = C + F_0) \\W_k &= (C + F_k)^{-1}U_k \\U_{k+1} &= U_k - CW_k + E_k \\X_k &= W_0 + W_1 + \cdots + W_k, \text{ for } k = 0, 1, 2, \dots\end{aligned}$$

The goal of this talk is to show that the above extended iterative refinement algorithm converges by providing a theorem that we called the theorem of convergence of the (extended) iterative refinement. The convergence of the iterative refinement is a central issue when solving ill conditioned linear systems. To compute the accurate solution  $x = A^{-1}b$  of an ill conditioned linear system  $Ax = b$  we use the Schur aggregation method and the Sherman-Morrison-Woodbury (SMW) formula  $A^{-1} = C^{-1} + C^{-1}U(I - V^HC^{-1}U)^{-1}V^HC^{-1}$ . The Schur aggregate  $S = I - V^HC^{-1}U$  is computed using the extended iterative refinement. The talk will also cover the notion Additive Preconditioner  $UV^H$  and when the  $A$ -modification  $C = A + UV^H$  is well conditioned. (Received September 25, 2012)