We study the second order semi-linear hyperbolic equation on bounded domain in $\mathbb{R}^d, d = 2, 3$ with smooth boundary given by

$$u_{tt} - \nabla \cdot (\nabla u) + f(u) = g \quad \text{with} \quad |f'(u)| \leq C(1 + |u|^p) \quad 0 \leq p \leq \frac{2}{d-2}.$$ 

We prove that the semidiscrete method conserves the energy. We also establish the improved $L^2$-error estimates for our method in both semidiscrete and fully discrete schemes, using the Sobolev-Poincaré inequality and the Gronwall inequality.

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