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The Matrix Lie Algebra Isomorphism problem asks whether two Lie algebras of  $n \times n$  matrices, given by bases, are conjugate by an invertible  $n \times n$  matrix. This problem arises naturally in Geometric Complexity Theory (GCT) in attempting to understand the orbits of symmetry-characterized functions such as the permanent and determinant from a computational perspective. (A function is symmetry-characterized if it is the only function that is stabilized by its stabilizer.) We show that Matrix Isomorphism for abelian diagonalizable matrix Lie algebras is at least as hard as Graph Isomorphism, and for semisimple matrix Lie algebras is equivalent to Graph Isomorphism. The complexity of Graph Isomorphism, in turn, is a long-standing open question. For the special cases of Lie algebras that arise from the determinant or matrix multiplication, we show that the Matrix Lie Algebra Isomorphism problem can be solved in polynomial time. This leads to an efficient computational understanding of these orbits, for example: testing whether a function  $f$  lies in the orbit of the determinant can be done in polynomial time. This latter connection relies crucially on the symmetry-characterization of the determinant. We also discuss more generally the role of symmetry-characterization in GCT. (Received September 24, 2012)