Let $G$ be an abelian group of rank 2 and order $N$, let $M_p(G)$ be the number of elliptic curves over the finite field $\mathbb{F}_p$ with group of points isomorphic to $G$. We study in this talk the average of $M_p(G)$ over the prime fields $\mathbb{F}_p$, in particular how the average varies with the structure of the group $G$. We find that this variation is governed by the Cohen-Lenstra Heuristics, which predict that random abelian groups occur with probability weighted by $\#G/\#\text{Aut}(G)$ where $\text{Aut}(G)$ is the number of elements of the automorphism group of $G$. This variation can also be seen when we forget the group structure, and look at the average number of curves with a fixed number of points over $\mathbb{F}_p$.

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