

1086-G5-1698      **Karsten K. Schmidt\*** (kschmidt@fh-sm.de). *Teaching Matrix Algebra with Magic Squares.*

A magic square of order  $n$  is a square arrangement of  $nn$  real numbers, such that the sums of the elements in each row, column, and diagonal are equal to a constant  $s$ , its magic sum. If an  $n \times n$  matrix  $M$  denotes a magic square, and  $\mathbf{j}$  denotes an  $n \times 1$  vector of ones, the following activities can be carried out in class (if possible, using technology to simplify calculations): computing the matrix product  $M\mathbf{j}$  and comparing it to the scalar product  $s\mathbf{j}$  to check whether the  $n$  row sums are indeed equal to  $s$ ; computing the trace of  $M$  to check whether the sum of the elements of the main diagonal is equal to  $s$ ; reconsidering the equation  $M\mathbf{j} = s\mathbf{j}$  to discover that  $s$  is one of the eigenvalues, and  $\mathbf{j}$  an associated eigenvector, of  $M$ . Any  $3 \times 3$  magic square can be written as the sum of two matrices,  $M = sG + N$ , where  $G = 1/3J$  ( $J = \mathbf{j}\mathbf{j}'$  denotes the  $3 \times 3$  matrix of ones), and also  $N$  has a simple structure defined by only two real numbers. The matrices  $G$ ,  $N$ , and  $M$  provide good examples to compute the trace, determinant, rank, and eigenvalues, and investigate the connections between them. A further interesting activity is to compute the (Moore-Penrose) inverse of  $M$ , and investigate whether it is also magic. The Lo-Shu magic square (4,9,2;3,5,7;8,1,6) will be the example used in the presentation. (Received September 24, 2012)