

1086-VN-1463 **Shaun Sullivan***, 3285 Cypress Legends Cir., #903, Fort Myers, FL 33965-6565. *Fine Ballot Numbers*.

The ballot numbers count paths that stay weakly above the diagonal $y = x$, start at the origin, and take steps from the set $\{\uparrow, \rightarrow\} = \{u, r\}$. Those ending on the diagonal can be viewed as Dyck paths. The Fine Numbers count Dyck paths with no hills, that is paths with no ur on the diagonal. We consider ballot paths with no hills, and ballot paths with k hills. Using Finite Operator Calculus, we can find a formula to enumerate ballot paths with k hills. We also generalize the result that the ratio of Fine numbers to Catalan numbers approaches $\frac{4}{9}$ to Dyck paths with k hills.

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