We show that for any finite Galois extension $K$ of the rational numbers $\mathbb{Q}$, there are infinitely many Carmichael numbers composed solely of primes for which the associated class of Frobenius automorphisms coincides with any given conjugacy class of $\text{Gal}(K|\mathbb{Q})$. This result implies that, for every natural number $n$, there are infinitely many Carmichael numbers of the form $a^2 + nb^2$ with $a, b$ integers. (Received September 24, 2012)