The Birkhoff pointwise ergodic theorem says that in a dynamical system, the ergodic averages

$$\frac{1}{N} \sum_{i<N} f(T^i x)$$

converge to a limit for all $x$ outside a set of measure 0. From the perspective of algorithmic randomness, it is natural to place computability restrictions on $T$ and $f$ and investigate the nature of the exceptional points.

On the dynamical system side, the problem depends significantly on whether the system is ergodic or not. (That is, whether the space can be decomposed into two disjoint sets of positive measure so that, up to measure 0, the sets are constant under $T$.) The relationship between ergodic and non-ergodic systems turns out to be closely analogous to the relationship between computable and computably enumerable functions. (Received September 13, 2013)