We give an overview of earlier results and recent developments in our investigation of the Turing degree spectra of structures, relations, automorphisms, and sets of relations. The degree spectrum of a countable structure $\mathcal{A}$ is the set of all degrees of the atomic diagrams of the isomorphic copies of $\mathcal{A}$. The degree spectrum of an additional relation $R$ on a computable structure $\mathcal{B}$ is the set of Turing degrees of the images of $R$ under all isomorphisms from $\mathcal{B}$ onto computable structures. While many degree spectra of relations have upper bounds, all nontrivial degree spectra of structures are closed upward in the Turing degrees. The automorphism degree spectrum of a computable structure is the set of degrees of its nontrivial automorphisms. While no automorphism degree spectrum consists of exactly two incomparable degrees, various sets of Turing degrees, including many upper cones, can be realized as automorphism degree spectra. We also consider a family of natural relations on a computable structure and investigate their degree spectrum. Our main example is the set of orders on a computable group. (Received September 15, 2013)