A map is a cellular embedding of a graph into a surface and a regular map is a highly symmetric map like five Platonic solids. A Cayley map is a cellular embedding of a Cayley graph into an orientable surface having the same cyclic rotation of generators around each vertex. Regular Cayley maps on a given group $G$ are related to skew morphisms of $G$ satisfying some conditions. Using these relations, we will consider several properties of regular Cayley maps on some groups. Furthermore, we give a talk about some recent results related to regular Cayley maps on dihedral groups, for example, classifications of reflexible regular Cayley maps and regular Cayley maps having as many as possible different power values. (Received September 12, 2013)