We study a variant of the family of Johnson graphs, the Full-Flag Johnson graphs. We show that Full-Flag Johnson graphs are Cayley graphs on $S_n$ generated by certain classes of permutations, and that they are in fact generalizations of permutahedra. We derive some results about the adjacency matrices of Full-Flag Johnson graphs and apply these to the set of permutahedra to deduce part of their spectra. (Received September 15, 2013)