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**William R. Pulleyblank\*** ([william.pulleyblank@usma.edu](mailto:william.pulleyblank@usma.edu)), United States Military Academy, Department of Mathematical Sciences, West Point, NY 10996. *Optimal linear arrangements and graceful labelings of graphs.*

A graceful labeling of a tree with  $n$  vertices is a labeling of its vertices with  $\{1, 2, \dots, n\}$  such that each vertex gets a distinct label, and such that, for each edge, the absolute difference between the labels on its endpoints is distinct. Forty-six years ago it was conjectured that every tree has a graceful labeling. This conjecture, called the Ringel-Kotzig conjecture, is still unresolved.

The Optimal Linear Arrangement problem is closely related. It again requires labeling the vertices of an  $n$  vertex graph with  $\{1, 2, \dots, n\}$  and then defining the label of each edge to be the absolute difference between the labels on its endpoints. The objective of this problem is to find such a vertex labeling for which the sum of the derived edge labels is minimum. Yossi Shiloach (1979) gave a polynomially bounded algorithm for solving this problem in trees, but which has not yet been successfully generalized to series-parallel graphs.

We discuss mixed integer programming formulations of these problems which seem to provide effective means of solving these problems for “moderately” sized graphs (dozens but not hundreds of vertices). This also leads to a strengthened version of the Ringel-Kotzig conjecture which, to our knowledge, is not resolved. (Received September 17, 2013)