Polynomial-Generated Orthogonal Latin Squares and Hypercubes.

The search for sets of mutually orthogonal latin squares (MOLS) at non-prime powers has proven a confounding question. When we consider latin squares of order $n$ (that is, $n \times n$ arrays on $n$ symbols where no symbol is repeated in a row or column) which have $n$ a prime power, it is simple to construct examples via polynomials over finite fields. And if we require sets of such squares to be mutually orthogonal (that is, when superimposed each pair of symbols occurs exactly once), we can find sets of such polynomials that attain the theoretical maximum: where $N(n)$ is the maximum size of such a set of squares of order $n$, $N(n) \leq n - 1$. But when $n$ is not a prime power, there is no finite field of order $n$ and things become more difficult. Although values for $N(n)$ are not determined for non-prime powers (except $N(6) = 1$) in this talk we explore what happens when we require the squares to be generated by polynomials over finite rings. In this case we can determine a maximum size of set of MOLS for all $n$. We also extend our reach to higher-dimensional arrays, and to frequency squares. (Received September 17, 2013)