A phased matroid is a combinatorial abstraction of a finite set of vectors in complex space. The phased matroid is a tool for keeping track of some of the geometric information of the set of vectors. Just as complex numbers are an extension of real numbers, phased matroids are an extension of oriented matroids, which is a well studied field.

The realization space of an oriented (resp., phased) matroid is the space of vector arrangements in $\mathbb{R}^n$ (resp., $\mathbb{C}^n$) that correspond to oriented (resp., phased) matroid, modulo a change of coordinates. The realization space of any rank 2 oriented matroid is contractible. According to Mnëv’s Universality Theorem, the realization spaces of oriented matroids with rank greater than or equal to 3 can be as complicated as any semi-algebraic variety. In fact, even uniform rank 3 oriented matroids can have a realization space homeomorphic to an (open) semi-algebraic variety.

In contrast, for uniform phased matroids which are not essentially oriented, the realization space is homeomorphically equivalent to $\mathbb{R}^{n-1}$ (where $n$ is the number of elements of the phased matroid). (Received September 18, 2013)