1096-05-320 **Jie Ma*** (majiemath@gmail.com) and **Humberto Naves**. *Maximizing the number of proper colorings of graphs.* Preliminary report.

We study an old problem of Linial and Wilf to find the graphs with n vertices and m edges which maximize the number of proper q-colorings of their vertices. In a breakthrough paper, Loh, Pikhurko and Sudakov reduced the problem to an optimization problem and solved asymptotically for many ranges of parameters. We prove the following structural result which tells us how the solution of the optimization looks like: for any instance, the optimization problem always has a solution which corresponds to either a complete multipartite graph or a graph obtained from complete multipartite graph by removing edges of two bipartite subgraphs.

We then apply this structural result of optimal graphs to general instances, including a conjecture of Lazebnik from 1989 which asserts that for any $q \geq s \geq 2$, the Turán graph $T_s(n)$ has the maximum number of q-colorings among all graphs with the same number of vertices and edges. We disprove this conjecture by providing infinity many counterexamples (s, q) for $s + 7 < q < 2s - 3$. On the positive side, we show that when $q \geq \Omega(s^2)$ the Turan graph $T_s(n)$ indeed achieves the maximum number of q-colorings.

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