Michael R. DiPasquale* (dipasqu1@illinois.edu), Department of Mathematics, 1409 W. Green St., Urbana, IL 61801. Lattice-Supported Splines on Polytopal Complexes.

The real vector space $C^r_k(P)$ of piecewise polynomial functions (splines) of smoothness $r$ and degree $\leq k$ on a pure $n$-dimensional polytopal complex $P \subset \mathbb{R}^n$ is of fundamental interest in approximation theory and numerical analysis. In the case where $P$ is a simplicial complex and $k \geq 3r + 2$ there is a basis for $C^r_k(P)$ which is ‘locally supported’ in the sense that each spline in the basis vanishes outside the star of a vertex [Ibrahim-Schumaker]. Such a basis is called a star-supported basis.

We show that there is an analog of star-supported bases for polyhedral complexes, which we call lattice-supported bases of $C^r_k(P)$. Each spline in a lattice-supported basis vanishes outside of a complex $P_W$ which is associated to a flat $W$ of the intersection lattice of interior codimension 1 faces of $P$. In particular we show $C^r_k(P)$ has a lattice-supported basis for $k \gg 0$. In the planar case we conjecture a lower bound on $k$ for when $C^r_k(P)$ has a lattice-supported basis. This is closely related to the question of finding $\dim_{\mathbb{R}} C^r_k(P)$ addressed in [Alfeld-Schumaker] and [McDonald-Schenck]. (Received September 11, 2013)