A classical theorem in number theory states that a positive integer $z$ can be written as the sum of two squares if and only if all prime factors $q$ of $z$ with $q \equiv 3 \pmod{4}$ have even exponent in the prime factorization of $z$. One can consider a minor variation of this theorem by not allowing the use of zero as a summand in the representation of $z$ as the sum of two squares. Viewing each of these questions in $\mathbb{Z}_n$, the ring of integers modulo $n$, we investigate which integers $n \geq 2$ are such that every $z \in \mathbb{Z}_n$ can be written as the sum of two squares in $\mathbb{Z}_n$. (Received September 11, 2013)