Local models are schemes which are intended to model the étale-local structure of integral models of Shimura varieties. Pappas and Zhu have recently given a general group-theoretic definition of local models with parahoric level structure, valid for any tamely ramified group, but it remains an interesting problem to characterize the local models, when possible, in terms of an explicit moduli problem. In the setting of local models for ramified, quasi-split $GU_n$, work towards an explicit moduli description has been done by Pappas and Pappas–Rapoport. We show that the conditions they formulate are not strong enough to cut out the (flat) local model in general, and we propose a general strengthening of their moduli problem. We furthermore show that our strengthened moduli problem does characterize the local model in the case of signature $(n-1,1)$ and a certain special maximal parahoric level structure, where Pappas and Rapoport’s conditions alone are not strong enough to do so. (Received September 16, 2013)