Groupoids at once generalize groups, group actions, equivalence relations, and group bundles. In his 1980 thesis, Renault explained how to construct a $C^*$-algebra $C^*(G, \omega)$ out of a groupoid $G$ and a 2-cocycle $\omega \in Z^2(G, \mathbb{T})$. These twisted groupoid $C^*$-algebras answer many questions about the structure of other $C^*$-algebras, and their $K$-theory gives us information about $D$-branes in string theory.

When $G$ is a group, Echterhoff et al. proved in 2010 that in many cases, a homotopy $\{\omega_t\}_{t \in [0,1]}$ of 2-cocycles on $G$ leaves the $K$-theory groups of the twisted group $C^*$-algebras invariant:

$$K_*(C^*(G, \omega_0)) \cong K_*(C^*(G, \omega_1)).$$

We investigate the extent to which this $K$-theoretic invariance extends to the world of groupoids.

We have expanded Echterhoff et al.'s result to the case of transformation groups $G \ltimes X$; using different techniques, inspired by a 2012 result of Kumjian et al., we also show that a homotopy of 2-cocycles on the groupoid $\mathcal{G}_\Lambda$ associated to a $k$-graph $\Lambda$ induces an isomorphism

$$K_*(C^*(\mathcal{G}_\Lambda, \omega_0)) \cong K_*(C^*(\mathcal{G}_\Lambda, \omega_1)).$$

This result suggests applications to the classification of $k$-graph $C^*$-algebras. (Received September 16, 2013)