In the "Categorical Framework for the Study of Singular Spaces" W. Fulton and R. MacPherson describe how bivariant theories can be used to study singular topological spaces. We modify and extend their approach to construct a bivariant theory that connects singular homology of quasi-projective schemes to cohomology with support for smooth pairs of schemes. This general construction is consistent with the known cases; for example, it connects singular Chow groups $CH^*(X)$ with the Operational Chow Groups for closed embeddings, $A^*(X \hookrightarrow Y)$.

We describe the most general type of 'pull-back' and 'push-forward' maps and list the conditions required for the existence of both these structures on a given theory. Moreover, our construction gives a compatible structure of higher Chern classes, fact that allows us to study the problem of a general Riemann-Roch theorem for singular homologies. (Received September 08, 2013)