Building Toward a “Twisted” Theta Correspondence. Preliminary report.

The local theta lift over a $p$-adic field takes a pair of reductive groups $G := G(F)$ and $G' = G'(F)$ (with $F$ a $p$-adic field) along with a symplectic vector space $V$ over $F$ for which $G \times G' \subset H := \text{Sp}(V)$. There exists covering groups $\tilde{G}$, $\tilde{G}'$, and $\tilde{H}$ and a representation $\omega$ of $\tilde{H}$ so that its restriction to $\tilde{G} \times \tilde{G}'$ decomposes “nicely”. Representations $\pi \boxtimes \pi'$ appearing in this restriction are called theta lifts of each other.

In our talk, we discuss some results regarding theta lifts when $G$ is a symplectic group and $G'$ is an orthogonal group for an odd-dimensional orthogonal space. Namely, we aim to sketch a proof of the famous theta dichotomy conjecture. The proof relies heavily on the Rankin-Selberg-type doubling integral of Piatetski-Shapiro and Rallis. From there, We will introduce on-going research that defines the “twisted” doubling integral and attempt to compute their $L$-functions as well as interpret them in the theta correspondence framework. (Received September 13, 2013)