We consider the eigenvalues of the magnetic Schrödinger operator on a quantum graph as functions of the magnetic potential. We establish a simple relation between the Morse index of the magnetic eigenvalue and the number of zeros of the corresponding non-magnetic eigenfunction. This highlights an intricate relationship between zeros of an eigenfunction and the stability of the corresponding eigenvalue under magnetic perturbation.

In particular, let \( \{\sigma_j\}_{j=1}^\beta \) be a set of generators of the fundamental group of a quantum graph \( \Gamma \). The eigenvalues of the magnetic Schrödinger operator may be considered as functions of the magnetic flux \( \alpha = (\alpha_1, \ldots, \alpha_\beta) \) where \( A(x) \) is the magnetic potential on \( \Gamma \) and

\[
\alpha_i = \oint_{\sigma_j} A(x) \, dx.
\]

Let \( \psi \) be the \( n \)-th eigenfunction of the ordinary Schrödinger operator (no magnetic potential) and assume that \( \psi \) is non-zero on the vertices of \( \Gamma \). Let \( \phi \) denote the number of internal zeros of \( \psi \) on \( \Gamma \). We demonstrate that \( (0, \ldots, 0) \) is a non-degenerate critical point of \( \lambda_n(\alpha) \) with Morse index equal to the nodal surplus of \( \psi \), which is \( \phi - (n - 1) \). (Received September 13, 2013)