THM: The solutions of the equation

\[((y'')^{-2/3})''' = 0\]

are precisely the non-degenerate conic sections.

We shall give two old, elementary and comprehensible proofs.

This theorem appears in Sylvester (1886), although Cartan (1937) attributes it to Halphen (1870’s). The essentially equivalent version

\[y''(40(y''')^3 - 45y'''y''y''' + 9(y'')^2y''') = 0,\]

which is valid for non-vertical lines as well, goes back to Monge (1809). More sophisticated interpretations of this result have been given by Lascoux (2006). (Received September 15, 2013)