We study solutions of $q$-difference Painlevé equations in an asymptotic limit. The specific equation we consider is a $q$-difference version of the first Painlevé equation ($qP1$) associated with rational surface of type $A_1^{(1)}$. We show that there are four families of almost stationary solutions in the limit as the independent variable approaches infinity, deduce their formal series expansions and study their convergence.

In the divergent case, we show that there exist true analytic solutions asymptotic to such series in a domain that contains all $q$-iterates of a given initial domain. Our results show that the solution is unstable in the space of solutions. The method, while demonstrated for $qP1$, is also applicable to other $q$-difference Painlevé equations and we suggest that the corresponding unstable solutions, which we call quicksilver solutions, should also exist for these other equations. (Received September 04, 2013)