Walter Craig* (craig@math.mcmaster.ca), Fields Institute, 222 College Street, Toronto, Ontario M5T 3J1, Canada. On the size of the Navier - Stokes singular set.

Consider the hypothetical situation in which a weak solution $u(t, x)$ of the Navier-Stokes equations in three dimensions develops a singularity at some singular time $t = T$. It could do this by a failure of regularity, or more seriously, it could also fail to be continuous in the strong $L^2$ topology. The famous Caffarelli Kohn Nirenberg theorem on partial regularity gives an upper bound on the Hausdorff dimension of the singular set $S(T)$. We study microlocal properties of the Fourier transform of the solution in the cotangent bundle $T^\ast(R^3)$ above this set. Our first result is that, if the singular set is nonempty, then there is a lower bound on the size of the wave front set $WF(u(T,.))$, namely, singularities can only occur on subsets of $T^\ast(R^3)$ which are sufficiently large. Furthermore, if the solution is discontinuous in $L^2$ we identify a closed subset $S'(T)$ of $S(T)$ on which the $L^2$ norm concentrates at this time $T$. We then give a lower bound on the microlocal manifestation of this $L^2$ concentration set, which is larger than the general lower bound above. An element of the proof of these two bounds is a global estimate on weak solutions of the Navier-Stokes equations which have sufficiently smooth initial data. (Received September 08, 2013)