I will discuss several related isoperimetric inequalities for Sobolev functions which extremize the ratio $\frac{\|\nabla u\|_{L^2}}{\|u\|_{L^p}}$, where $1 \leq p < \frac{2n}{n-2}$. On of these inequalities generalizes a family of inequalities due to Payne, Chiti, et. al, and another generalizes the classical Schwarz Lemma from complex analysis. Curiously, one inequality has a nice interpretation as a classical isoperimetric inequality for a certain conformally flat metric with singularities. (Received September 09, 2013)