It was the fractal nature of the spectrum of the almost Mathieu operator, known as “Hofstadter’s butterfly,” which sparked the interest in quasi-periodic Schrödinger operators. They describe the influence of a magnetic field on the electrons in a crystal.

In physics the term “Cantor spectrum” is often used for appearance of singular continuous (SC) spectrum. Crucial in capturing this phenomenon is the Lyapunov exponent (LE) of the matrix cocycles associated with the Schrödinger equation. For analytic potentials and typical magnetic frequency, SC spectrum is known to occur only on the set of zero LE. As the same set also supports the absolutely continuous (AC) spectrum (“scattering states”), distinction of the two contributions is needed.

We discuss methods to localize the zero LE regime which explicitly distinguish between contributions from AC- (“subcritical regime”) and singular spectrum (“critical regime”). Applications include the self-dual, isotropic extended Harper’s model where zero LE with purely SC spectrum could be proven, verifying a conjecture of Thouless. For a potential given by a trigonometric polynomial, a criterion for subcritical behavior is presented. The latter is based on upper bounds of the LE of the complexified Schrödinger cocycle. (Received September 13, 2013)