The Poincaré-Birkhoff theorem states that an area-preserving twist homeomorphism of the annulus has at least 2 fixed points. In this paper, we provide a combinatorial proof of the theorem which is similar in spirit to the proof of the Brouwer Fixed Point theorem involving Sperner’s Lemma. We define and follow a discrete ‘pushing’ path, which must end in a nonessential loop of winding number 1, forcing the existence of a fixed point. The process is repeated, following pushing paths from a small neighborhood around the first fixed point. Either one path forms a nonessential loop as before, or a combination of these paths may be pieced together to form such a loop, establishing a second fixed point. Our method is constructive, so we end by discussing its application to computational algorithms. (Received September 16, 2013)