We study maps of the real projective plane with the property that they are quotient maps of analytic, or rational, maps of the sphere; these are called dianalytic maps. We start by describing topological work done jointly with Sue Goodman showing how to construct these dianalytic maps via sphere maps. We next discuss measure theoretic properties of these maps with a view to identifying natural measures on the Julia sets, and give some results on one-sided Bernoulli and ergodic measures. We show for example, why the maps are not one-sided Bernoulli with respect to conformal measure and describe the unique measure of maximal entropy. We give examples where the Julia set is the projective plane and discuss ergodic properties of these maps. (Received September 04, 2013)