Consider the following nonlinear variation on the Fibonacci sequence: $a_0 = 1$, $a_1 = 1$, and for $n \geq 2$,

$$a_n = \begin{cases} \frac{a_{n-1}+a_{n-2}}{5}, & \text{if this is an integer}, \\ a_{n-1} + a_{n-2}, & \text{otherwise}. \end{cases}$$

This sequence proceeds 1, 1, 2, 3, 1, 4, 1, 1, ..., and is periodic of length 6. This sequence is quite special. With initial conditions $a_0 = 1$, $a_1 = 3$, the resulting sequence grows without bound. Change the 5 in the denominator and the pattern changes as well. It appears that 2, 3, 5 are the only (positive) denominators which allow periodic sequences. To generalize, fix integers $P, Q$ and a rational number $r$. We investigate periodic behavior for sequences of the form

$$a_n = \begin{cases} r(Pa_{n-1} - Qa_{n-2}), & \text{if this is an integer}, \\ Pa_{n-1} - Qa_{n-2}, & \text{otherwise}. \end{cases}$$

(Received September 13, 2013)