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**John R. Greene\*** (jgreene@d.umn.edu), Department of Mathematics and Statistics, 140 Solon Campus Center, 1117 University Drive, Duluth, MN 55812. *A number theoretic twist on second order recurrence relations.* Preliminary report.

Consider the following nonlinear variation on the Fibonacci sequence:  $a_0 = 1$ ,  $a_1 = 1$ , and for  $n \geq 2$ ,

$$a_n = \begin{cases} \frac{a_{n-1} + a_{n-2}}{5}, & \text{if this is an integer,} \\ a_{n-1} + a_{n-2}, & \text{otherwise.} \end{cases}$$

This sequence proceeds 1, 1, 2, 3, 1, 4, 1, 1,  $\dots$ , and is periodic of length 6. This sequence is quite special. With initial conditions  $a_0 = 1$ ,  $a_1 = 3$ , the resulting sequence grows without bound. Change the 5 in the denominator and the pattern changes as well. It appears that 2, 3, 5 are the only (positive) denominators which allow periodic sequences. To generalize, fix integers  $P$ ,  $Q$  and a rational number  $r$ . We investigate periodic behavior for sequences of the form

$$a_n = \begin{cases} r(Pa_{n-1} - Qa_{n-2}), & \text{if this is an integer,} \\ Pa_{n-1} - Qa_{n-2}, & \text{otherwise.} \end{cases}$$

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