The Hierarchical Graph Laplacian Eigen Transform (HGLET) and Its Relatives for Data Analysis on Graphs and Networks.

The Hierarchical Graph Laplacian Eigen Transform (HGLET) is our new analysis tool for data measured on vertices of a given graph. It starts with computing the graph Laplacian eigenvectors of the whole graph. Then, using the Fiedler vector (i.e., the eigenvector corresponding to the second smallest eigenvalue), we partition the graph into two subgraphs. In each subgraph, we compute its own graph Laplacian eigenvectors, and we repeat this process recursively until each subgraph has a single vertex. This tree-structured set of eigenvectors contains a redundant set of orthonormal bases and allows us to extract the most suitable orthonormal basis for the task at hand (e.g., denoising) using the best-basis type algorithm. We will also describe two variants of the HGLET: the Haar-like HGLET and its smoother version called the Orthonormalized Hierarchical Fiedler Transform (OHFT). Both use only the Fiedler vector of each subgraph. The former binarizes each Fiedler vector according to its sign under the constraint of being orthogonal to the constant-valued vector while the latter applies the orthogonalization procedure to the computed Fiedler vector after each partition against all those previously computed. We will demonstrate their usefulness for various applications. (Received September 14, 2013)