Gideon Schechtman and I prove that if $X$ is a subspace of an $L_p$ space, $1 < p < 2$, and $\ell_p(\aleph_1)$ does not embed into $X$, then $X$ embeds into $L_p(\mu)$ for some finite measure $\mu$. We also give appropriate versions of this result for $\ell_p(\aleph)$ with $\aleph$ any uncountable cardinal. The $\aleph_1$ result complements a theorem proved by P. Enflo and H. P. Rosenthal 40 years ago; namely, that $\ell_p(\aleph_1)$, $1 < p < 2$, does not (isomorphically) embed into $L_p(\mu)$ with $\mu$ a finite measure.

This work is an outgrowth of an unsuccessful attempt to solve a problem left open by Enflo and Rosenthal; namely, whether $L_p(\mu)$ can have an unconditional basis when $1 < p \neq 2 < \infty$, the measure $\mu$ is finite, and the density character of $L_p(\mu)$ is $\aleph_1$. If the answer is yes, then the results of Enflo and Rosenthal would show that “$L_p(\{-1,1\})^{2^{\aleph_0}}$ has an unconditional basis” is undecidable. (Received September 15, 2013)