We prove that the spaces $\ell_p$, $1 < p < \infty$, $p \neq 2$, and all their infinite-dimensional subspaces do not admit equivalent almost transitive renormings. This answers a problem posed by Deville, Godefroy and Zizler in 1993. We obtain this as a consequence of a new property of almost transitive spaces with a Schauder basis, namely we prove that in such spaces the unit vector basis of $\ell_2^2$ belongs to the two-dimensional asymptotic structure and we obtain some information about the asymptotic structure in higher dimensions.

We also prove that the spaces $\ell_p$, $1 < p < \infty$, $p \neq 2$, have continuum different renormings with 1-unconditional bases each with a different maximal isometry group, and that every 1-symmetric space other than $\ell_2$ has at least a countable number of such renormings. On the other hand we show that the spaces $\ell_p$, $1 < p < \infty$, $p \neq 2$, have continuum different renormings each with an isometry group which is not contained in any maximal isometry group of a renorming of $\ell_p$. This answers a question of Wood. (Received September 11, 2013)