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Hugo J Woerdeman* (hugo@math.drexel.edu), Department of Mathematics, Drexel University, 3141 Chestnut Street, Philadelphia, PA 19104. *Determinantal representations of stable polynomials.*

A polynomial $p(z) = p(z_1, \dots, z_d)$ is called *stable* if $p(z) \neq 0$ for $z \in \overline{\mathbb{D}}^d$, where \mathbb{D} is the open unit disk in \mathbb{C} and $\overline{\mathbb{D}} = \mathbb{D} \cup \mathbb{T}$ is its closure. It is an open question whether every multivariable stable polynomial $p(z)$ with $p(0) = 1$ can be written as

$$p(z) = \det(I - KZ(z)),$$

where $Z(z)$ is a diagonal matrix with coordinate variables on the diagonal and K is a strict contraction. For one and two variable polynomials such a representation always exists; in one variable it is an easy consequence of the fundamental theorem of algebra, while in two variables it follows from a result by A. Kummert. As a variation, we show in this paper that independent of the number of variables every stable multivariable polynomial p , with $p(0) = 1$, has a determinantal representation

$$p(z) = \det(I - M(z)),$$

where $M(z)$ is a matrix valued rational function with $\|M(z)\| \leq 1$ and $\|M(z)^n\| < 1$ for $z \in \mathbb{T}^d$ and $M(az) = aM(z)$ for all $a \in \mathbb{C} \setminus \{0\}$. In fact, the existence of such a representation characterizes stable polynomials. (Received September 14, 2013)