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John D. Williams* (jwilliams@math.tamu.edu), Texas A&M, Department of Mathematics,
Mail Stop 3368, College Station, TX 77843-3368. *Analytic Function Theory for Operator-Valued
Free Probability.*

It is a classical result in complex analysis that the class of functions that arise as the Cauchy transform of probability measures may be characterized entirely in terms of their analytic and asymptotic properties. Such transforms are a main object of study in non-commutative probability theory as the function theory encodes information on the probability measures and the various convolution operations. In extending this theory to operator-valued free probability theory, the analogue of the Cauchy transform is a non-commutative function with domain equal to the non-commutative upper-half plane. In this paper, we prove an analogous characterization of the Cauchy transforms, again, entirely in terms of their analytic and asymptotic behavior. We further characterize those functions which arise as the Voiculescu transforms of \boxplus -infinitely divisible \mathcal{B} -valued distributions. As an immediate consequence this theorem, combined with an existing result of Popa and Vinnikov, may be used to produce analogues of the Nevanlinna representation for non-commutative functions with the appropriate asymptotics. In addition to this, we may define semigroups of completely positive maps associated to infinitely divisible distributions. (Received September 03, 2013)