The Centroidal Voronoi Tessellation (CVT) is an optimal configuration of points in convex domains. These tessellations are used in many areas from facility location problems to mesh generation. We extend the notion of a CVT from points to rigid shapes in two and three dimensions. Given a finite set of shapes, we optimize their location via translation and rotation by minimizing a suitable cost function. Differentiating the cost function reduces to differentiating the solution to a nonlinear PDE—the Eikonal Equation—with respect to its boundary condition (the zero contour). The CVT optimization problem for points is typically tackled using quasi-Newton methods and an iterative algorithm called Lloyd’s method—we will discuss extensions of both to the rigid shape case. The optimization problem for rigid shapes is challenging in part because integrals over the Voronoi regions generated by shapes must be calculated; these regions are non-convex and a priori unknown. The novelty of our algorithm is that the boundaries of the Voronoi regions are never explicitly calculated. We describe how the Eikonal Equation enters in the problem, and present detailed theoretical and numerical results. (Received September 16, 2013)