Blaschke’s Rolling Ball Property and Conformal Metric Ratios.

This research is focused on a generalization of Blaschke’s Rolling Ball Property that has been used to study various problems from mathematical morphology, image analysis, and smoothing. The purpose of this work was to characterize the closed sets in Euclidean space that satisfy a two-sided rolling ball property and to show that certain conformal metric ratios have a boundary value of one as an application. A closed set satisfies the two-sided rolling ball property provided it is possible to freely roll a ball with fixed radius inside and outside the closed set along its boundary. The main theorem proved by a geometric approach can be summarized as follows:

A non-empty and closed set has the two-sided rolling ball property if and only if it is an orientable $C^{1,1}$ smooth embedded submanifold, and there is a globally defined Lipschitz continuous unit normal vector field along it.

The size of a singular set was also studied in this work. Here, the singular set for a $C^1$ hypersurface is the set of points where the associated unit normal vector field is not differentiable. (Received August 22, 2013)