In joint work with J. M. Douglass, G. Pfeiffer, and G. Röhrle we refine a conjecture of Lehrer and Solomon on the structure of the Orlik-Solomon algebra of a finite Coxeter group $W$ and we relate it to the descent algebra of $W$. As a result, we claim that both the group algebra of $W$ and the Orlik-Solomon algebra of $W$ can be decomposed into sums of induced one-dimensional representations of element centralizers, one for each conjugacy class of elements of $W$. To date the conjecture has been verified for the symmetric groups and the dihedral groups, as well for all Coxeter groups of exceptional type other than the group of type $E_8$. We discuss the proof in the case of the symmetric and dihedral groups and time permitting, the computer implementation for the exceptional groups. (Received September 16, 2013)