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Conformally flat, constant $Q$-curvature metrics with isolated singularities. Preliminary report.

For $n \geq 5$ let $\Omega \subset \mathbb{R}^n$ be a domain with smooth boundary, and let $u \in C^\infty(\Omega)$ with $u > 0$. Then the conformally flat metric $g_{ij} = u^{\frac{4}{n-4}} \delta_{ij}$ has (fourth-order) $Q$-curvature given by

$$Q_g = \frac{4}{n-2} u^{-\frac{n+4}{n-4}} \Delta^2 u,$$

where $\Delta$ is the usual Laplace operator. The $Q$-curvature is an interesting quantity in conformal and Riemannian geometry, and it in some way generalizes scalar curvature. Also, the PDE one obtains by setting $Q_g$ to a positive constant is exactly the Euler-Lagrange equation to find the best constant in the Sobolev embedding $W^{2,2}(\Omega) \hookrightarrow L^{\frac{2n}{n-4}}$. Notice that the exponent $p = \frac{2n}{n-4}$ is exactly the exponent where the Sobolev embedding loses compactness, and so one should expect "bubbling" in the sense originally described by Uhlenbeck, leading to singular solutions.

I will discuss solutions to the PDE

$$\Delta^2 u = \frac{n(n-4)(n^2-4)}{16} u^{\frac{n+4}{n-4}}, \quad u > 0 \text{ in } B\setminus\{0\},$$

which give conformally flat metrics with constant positive $Q$-curvature in a punctured ball. In particular, I will describe some of the asymptotic behavior of $u(x)$ as $|x| \to 0$. (Received September 09, 2013)