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*Conformally flat, constant  $Q$ -curvature metrics with isolated singularities.* Preliminary report.

For  $n \geq 5$  let  $\Omega \subset \mathbf{R}^n$  be a domain with smooth boundary, and let  $u \in \mathcal{C}^\infty(\Omega)$  with  $u > 0$ . Then the conformally flat metric  $g_{ij} = u^{\frac{4}{n-4}} \delta_{ij}$  has (fourth-order)  $Q$ -curvature given by

$$Q_g = \frac{4}{n-2} u^{-\frac{n+4}{n-4}} \Delta^2 u,$$

where  $\Delta$  is the usual Laplace operator. The  $Q$ -curvature is an interesting quantity in conformal and Riemannian geometry, and it in some way generalizes scalar curvature. Also, the PDE one obtains by setting  $Q_g$  to a positive constant is exactly the Euler-Lagrange equation to find the best constant in the Sobolev embedding  $W^{2,2}(\Omega) \hookrightarrow L^{\frac{2n}{n-4}}$ . Notice that the exponent  $p = \frac{2n}{n-4}$  is exactly the exponent where the Sobolev embedding loses compactness, and so one should expect "bubbling" in the sense originally described by Uhlenbeck, leading to singular solutions.

I will discuss solutions to the PDE

$$\Delta^2 u = \frac{n(n-4)(n^2-4)}{16} u^{\frac{n+4}{n-4}}, \quad u > 0 \text{ in } \mathbf{B} \setminus \{0\},$$

which give conformally flat metrics with constant positive  $Q$ -curvature in a punctured ball. In particular, I will describe some of the asymptotic behavior of  $u(x)$  as  $|x| \rightarrow 0$ . (Received September 09, 2013)