We construct a combinatorial mechanism for proving cohomological vanishing results on certain classes of spaces, by means of a Mayer-Vietoris-type spectral sequence. The spaces include complex hyperplane complements, their De Concini-Procesi compactifications, complements of torus arrangements, and certain generalised moment-angle complexes. This builds on work of Davis, Januszkiewicz, Leary, and Okun. In particular, we generalize classical vanishing results originating in a paper of Esnault, Schechtman, and Vieweg. (Received September 16, 2013)