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**Xiangjin Xu\*** ([xxu@math.binghamton.edu](mailto:xxu@math.binghamton.edu)), Department of Mathematical Sciences, Binghamton University-SUNY, 4400 Vestal Parkway East, Binghamton, NY 13902. *New Heat Kernel Estimates on Riemannian Manifolds with Negative Curvature*. Preliminary report.

Apply the new Li-Yau type Harnack estimates for the heat equations on manifolds with  $Ric(M) \geq -K$ ,  $K \geq 0$ , which established by Junfang Li and the author [Advance in Mathematics 226(5) (2011), 4456-4491], we prove a new upper bound estimate for the heat kernel  $H(x, y, t)$  of manifolds with  $Ric(M) \geq -K$ ,

$$H(x, y, t) \leq A_K(t)V_x^{-1/2}(\delta(t))V_y^{-1/2}(\delta(t)) \exp \left[ -\frac{d^2(x, y)}{4t} + [1 + d^2(x, y)]B_K(t) \right],$$

where  $A_K(t), B_K(t) : [0, \infty) \rightarrow [0, \infty)$  are bounded functions, and  $\delta(t) \sim t$  as  $t \rightarrow 0$  and  $\delta(t) \sim 1$  as  $t \rightarrow \infty$ . While in the seminal work of Li-Yau [Acta Math. 156 (1986) 153-201.], the heat kernel upper bound estimates had  $\delta$ -loss:

$$H(x, y, t) \leq C(\delta, n)V_x^{-1/2}(\sqrt{t})V_y^{-1/2}(\sqrt{t}) \exp \left[ -\frac{d^2(x, y)}{(4 + \delta)t} + C_1\delta Kt \right],$$

where constant  $C(\delta, n) \sim \exp \left[ \frac{c_1}{\delta} \right]$  as  $\delta \rightarrow 0$ , due that there was non-sharp Harnack estimates on manifolds with negative curvature. Some new lower bound estimates of the heat kernel are also discussed. (Received August 26, 2013)