Tempered stable distributions were introduced in Rosiński 2007 as models that look like infinite variance stable distributions in some central region, but they have lighter (i.e. tempered) tails. Such models have found applications in a variety of areas including mathematical finance, biostatistics, computer science, and physics. We extend this class to allow for more variety in the tails. While some cases no longer correspond to stable distributions they serve to make the class more flexible, and, in certain subclasses, they have been shown to provide a good fit to data. To characterize the possible tails we give detailed results about finiteness of various moments, and we give necessary and sufficient conditions for the tails to be regularly varying. This last part allows us to characterize the domain of attraction to which a particular tempered stable distribution belongs. We then describe the weak limits of sequences of tempered stable distributions, and we conclude by characterizing the long and short time behaviour of their associated Lévy processes. (Received September 11, 2013)