Analyzing the Errors of an Extension of Wilkinson’s Iterative Refinement or Improvement Algorithm.

To find an accurate solution of an ill conditioned linear system $Ax = b$, we use the additive preconditioning $A \rightarrow C = A + UV^H$ for the preconditioner $UV^H$ of a smaller rank $r$ and the Schur aggregation technique to reduce the computation of $x = A^{-1}b$ to the computation of the Schur aggregate $S = I_r - V^H C^{-1}U$. The Schur aggregation is a process of transforming the linear system $Ax = b$ into better conditioned linear systems of smaller sizes, with well conditioned matrices $V^H C^{-1}$, $C^{-1} U$, $S = I_r - V^H C^{-1} U$ using the Sherman-Morrison-Woodbury (SMW) formula $A^{-1} = (C - UV^H)^{-1} = C^{-1} + C^{-1} U (I_r - V^H C^{-1} U)^{-1} V^H C^{-1}$. We find $S$ by computing $W = C^{-1} U$ using an extension of Wilkinson iterative refinement or improvement algorithm. Some steps of the algorithm are computed error free and other steps are computed with errors that need to be evaluated in order to determine the accuracy of the algorithm. In this presentation we will discuss the upper bound of the forward error of the algorithm to determine if its solution $W = C^{-1} U$ can be considered accurate enough. (Received September 17, 2013)