We present and analyze a linear programming algorithm based on replacing the non-negative orthant with larger quadratic cones. For each quadratic relaxation that has an optimal solution, there naturally arises a parameterized family of quadratic cones for which the optimal solutions create a path leading to the linear program’s optimal solution. We show that this path can be followed efficiently, thereby resulting in an algorithm whose complexity matches the best bounds proven for interior-point methods. Furthermore, this algorithm can be extended for the larger classes of semidefinite programming and hyperbolic programming problems, with similar iteration-complexity bounds. (Received September 17, 2013)