On the reconciliation of different non-equivalent definitions designed for the same concept, in the context of limits for two-variable functions.

Two definitions are considered for the limit at a point $P$ of a function mapping (a subset of) the real plane into the real numbers. For the first, convergence at $P$ is determined by establishing that for any real interval centred at the proposed limit, a disc centred at $P$ (lacking $P$) is mapped into that interval. For the second, convergence at the point $P$ is determined by the existence and consistency of the limits at $P$ over all half-lines emanating from $P$. Educational issues that arise are: Geometric interpretation of the limiting processes; parameterization (to utilize the definition of a limit for the one-variable case); grounds to prefer one definition over another; what it does mean for definitions to be equivalent; would students consider and determine that the two definitions above are not equivalent? I will present a teaching sequence guiding students to adapt the second definition to become equivalent to the first. For this, the notion of the ‘greatest’ $\delta$ given a particular $\epsilon$ for 1-variable functions is required; this notion is modeled in terms of sets as the ‘greatest’ $\delta$ may not exist (so suprema come in). (Received September 18, 2013)