Sparse signals can be succinctly represented by certain low-dimensional linear sketches with applications in compressive sensing, data streaming and graph-sketching, among others. Recently, structured sparsity has emerged as a promising new tool for reducing sketch size and improving recovery. Existing work on sketching structured sparse signals requires dense sketching matrices. On the other hand, sparse sketching matrices, usually from expanders, are computationally much more efficient, easier to store and apply in recovery. In this paper, we focus on model-based expanders, that is expanders that capture a given structure sparsity model, and show that they exist for a larger class of models than previously considered. We present the first polynomial time algorithm for recovering structured sparse signals from low-dimensional linear sketches obtained via sparse matrices. The algorithm is guaranteed to yield signals with bounded recovery error and is quite easy to implement and customize for structured sparse models that are endowed with a projection operator. As a result, we characterize a broad class of structured sparsity models that have polynomial time projection property. We also provide numerical experiments to illustrate the theoretical results in action. (Received September 17, 2013)