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Generalizing Zeckendorf's Theorem to f -decompositions.

A beautiful theorem of Zeckendorf states that every positive integer can be uniquely expressed as a sum of non-consecutive Fibonacci numbers. For sequences $\{G_n\}$ satisfying linear recurrence relations with nonnegative coefficients, there is a notion of a legal decomposition which again leads to a unique representation. The number of summands in the representations of $m \in [G_n, G_{n+1})$ converges to a Gaussian as $n \rightarrow \infty$.

Given a notion of legal decomposition, we ask if there exists a sequence $\{a_n\}$ such that every positive integer can be uniquely decomposed as a sum of terms from $\{a_n\}$. Let $f : \mathbb{N}_0 \rightarrow \mathbb{N}_0$. We say that if a_n is in an “ f -decomposition” of a number x , then the decomposition cannot contain the $f(n)$ terms immediately before a_n in the sequence. We prove that for any $f : \mathbb{N}_0 \rightarrow \mathbb{N}_0$, there exists a sequence $\{a_n\}$ such that every positive integer has a unique f -decomposition using $\{a_n\}$. If f is periodic, then the unique increasing sequence $\{a_n\}$ induced by f satisfies a linear recurrence relation. For some class of functions f , we prove that the number of summands in the f -decomposition of integers in a suitable growing interval converges to a normal distribution. (Received August 23, 2013)