Properties of the \((a, b)\)-Fibonacci Sequence, Modulo \(m\).

For integers \(a\) and \(b\), we consider the \((a, b)\)-Fibonacci sequence \(F\) defined by \(F_0 = 0\), \(F_1 = 1\), and \(F_n = aF_{n-1} + bF_{n-2}\). \(F\) (mod \(m\)) is periodic with period denoted \(\pi(m)\). The rank of \(F\) (mod \(m\)), denoted \(\alpha(m)\), is the least positive \(r\) such that \(F_r \equiv 0 \pmod{m}\), and the order of \(F\) (mod \(m\)), denoted \(\omega(m)\), is \(\pi(m)/\alpha(m)\). We pull together results on \(\pi(m)\), \(\alpha(m)\), and \(\omega(m)\) from the classic case \(a = 1\), \(b = 1\), and generalize them to accommodate arbitrary integers \(a\) and \(b\). Matrix methods are used to provide elementary proofs. (Received September 17, 2013)